Problem: I would like to better understand how ADMB estimates standard errors in random-effects models.

If I have n years of data, and assume annual survival probability (for purposes of a simple example) $\mu_s + \epsilon_i, i = 1, \ldots, n$ of a group of animals to be distributed as $N(\mu_s, \sigma_s^2)$, and I fit the model with ADMB, I obtain an estimate $\hat{\mu}_s$ of μ_s and its associated standard error. I also obtain an estimate $\hat{\sigma}_s^2$ of σ_s^2, and its associated standard error.

Question 1: When computing the standard error of $\hat{\mu}_s$, does ADMB use the estimate of process error, $\hat{\sigma}_s^2$, as in

$$\hat{Var}(\hat{\mu}_s) = E(Var(\hat{\mu}_s|\mu_s)) + Var(E(\hat{\mu}_s|\mu_s))$$

and estimate this with

$$\hat{Var}(\hat{\mu}_s|\mu_s) + \hat{\sigma}_s^2?$$

(Is this what's reported in the .std file?)

Question 2: Similarly, if I estimate abundance N_1 directly as $\hat{N}_1$, does ADMB use the estimate of process error if I compute $\hat{N}_2 = \hat{N}_1(\hat{\mu}_s + \hat{\epsilon}_1)$, as in

$$\begin{aligned}
&Var(\hat{N}_2) = Var(\hat{N}_1(\hat{\mu}_s + \hat{\epsilon}_1)) \approx \\
&(\hat{\mu}_s + \hat{\epsilon}_1)^2 \, Var(\hat{N}_1) + \hat{N}_1^2 Var(\hat{\mu}_s) + \hat{N}_1^2 Var(\hat{\epsilon}_1) + \\
&2\hat{N}_1(\hat{\mu}_s + \hat{\epsilon}_1) Cov(\hat{N}_1, \hat{\mu}_s) + 2\hat{N}_1 \hat{\mu}_s Cov(\hat{N}_1, \hat{\epsilon}_1) + 2\hat{N}_1^2 Cov(\hat{\mu}_s, \hat{\epsilon}_1)
\end{aligned}$$

where sampling error for $Var(\hat{N}_1)$ and $Var(\hat{\epsilon}_1)$ are estimated from the inverse-Hessian, $Var(\hat{\mu}_s)$ is estimated as above, and one of the covariance terms involving the random parameter is computed as

$$\begin{aligned}
&Cov(\hat{N}_1, \hat{\mu}_s) = E(Cov(\hat{N}_1, \hat{\mu}_s|\mu_s)) + Cov(E(\hat{N}_1|s), E(\hat{\mu}_s|\mu_s)) = \\
&\hat{Cov}(\hat{N}_s, \hat{\mu}_s) + Cov(N_1, \mu_s)
\end{aligned}$$

where the first term comes from the inverse-Hessian, and the second term would appear to depend on the specific model begin studied (probably zero in this case, since N_1 is assumed to be a fixed parameter)?